# Chapter 7

# Extension

## MARGINAL MEAN COMPARISONS WITHOUT HOMOGENEITY ASSUMPTION

It is often helpful in working with contrasts of marginal means to realize that in effect the factorial design is reduced to a one-way design when marginal means are being examined. Thus the same principles we developed there also apply here. This conceptualization is especially helpful if we are concerned about possible violations of homogeneity of variance. For example, suppose that we want to test the difference between Drug *X* and Drug *Y*, just as we did in the preceding section, except now we are unwilling to assume homogeneity of variance. We can express the contrast in terms of the six-cell means as

$$\psi = .5\mu_{11} + .5\mu_{21} - .5\mu_{12} - .5\mu_{22} + 0\mu_{13} + 0\mu_{23} .$$

For our data, $\hat{\psi}$ is given by

$$\begin{aligned} \psi &= .5\bar{Y}_{11} + .5\bar{Y}_{21} - .5\bar{Y}_{12} - .5\bar{Y}_{22} \\ &= .5\,(170) + .5\,(186) - .5\,(203) - .5\,(201) \\ &= -24. \end{aligned}$$

The corresponding sum of squares for the contrast can be found from Equation 4.30:

$$SS_{\psi} = E_{\mathrm{R}} - E_{\mathrm{F}} = (\hat{\psi})^2 / \sum_{j=1}^{ab} (c_j^2 / n_j) . \quad \text{(see 4.30)}$$

The $j$ subscript ranges from 1 to the number of cells in the original factorial design. For these data, the sum of squares equals

$$SS_{\psi} = (-24)^2 / \left[(.25/5) + (.25/5) + (.25/5) + (.25/5)\right] = 2880 .$$

Notice that this is precisely the value we obtained when we used Equation 7.48 (in the book) to calculate the sum of squares directly from the marginal means. As in the one-way design, the only difference that emerges, if we are unwilling to assume homogeneity, is that we do not use $MS_W$ as the error term. Instead, the separate variance approach uses an error term of the form

$$\text{denom} = \left( \sum_{j=1}^{ab} (c_j^2 / n_j) s_j^2 \right) \Bigg/ \sum_{j=1}^{ab} (c_j^2 / n_j) \, . \qquad \text{(see 4.33)}$$

In our example, the appropriate error term equals

$$\begin{aligned} \text{denom} &= \frac{(.25/5)(62.5) + (.25/5)(117.5) + (.25/5)(193.5) + (.25/5)(119.5)}{(.25/5) + (.25/5) + (.25/5) + (.25/5)} \\ &= 123.25. \end{aligned}$$

The resultant $F$ value is thus given by $F$ = 2,880/123.25 = 23.37. Assuming that we wish to use Tukey's HSD (or Honestly Significant Difference) to control $\alpha_{FW}$,[1] from Table 5.16, we can see that the appropriate critical value equals $(q_{0.5,\, b,\, df})^2/2$, where

$$df = \frac{\sum_{j=1}^{ab} (c_j^2 s_j^2 / n_j)^2}{\sum_{j=1}^{ab} \left[ (c_j^2 s_j^2 / n_j)^2 / (n_j - 1) \right]} \, . \qquad \text{(see 4.34)}$$

For our data, it turns out that $df$ = 14, or just over half the degrees of freedom we had when we assumed homogeneity. From Appendix Table 4, the critical $q$ value is 3.70 (remember that $b$ = 3, even though there are six individual cell means). The critical value for $F$ is then 6.84 (i.e., 3.70 squared and then divided by 2). As happened when we assumed homogeneity, the observed $F$ exceeds the critical value, so we can conclude that there is a difference in the population marginal means of Drug $X$ and Drug $Y$.

## NOTE

1. As discussed in Chapter 5, we would typically want to use Tukey's HSD to control $\alpha_{FW}$ if we are testing all pairwise comparisons or if we decided to test this particular pairwise comparison after having examined the data.